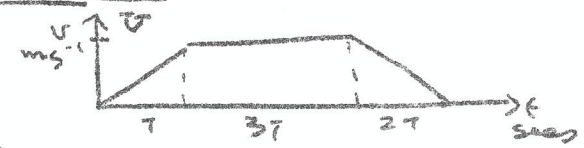
1978 [CL]



$|PQ| = \frac{1}{2}VT + 3VT + \frac{1}{2}(2)V$

But $V = 0 + aT = aT$.

$|PQ| = \frac{1}{2}aT^2 + 3aT^2 + aT^2$

$|PQ| = \frac{9}{2}aT^2$.

Same distance for second driver

Still $|PQ|$ = total distance $= \frac{9}{2}aT^2$

(i) Speed limit $3aT$.

? Can he reach $3aT$.

Accel to $3aT$.

$v = 3aT$.
$u = 0$.
$a = 2a$
$v^2 = u^2 + 2as_1$
$\Rightarrow (3aT)^2 = 0^2 + 4as_1$
$\Rightarrow 9a^2T^2 = 4as_1$
$\Rightarrow \frac{9aT^2}{4} = s_1$

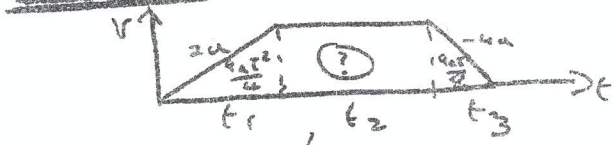
Decel from $3aT$

$u = 3aT$.
$v = 0$
$a = -4a$.
$v^2 = u^2 + 2as_3$
$0^2 = (3aT)^2 - 8as_3$
$\frac{9aT^2}{8} = s_3$

$s_1 + s_3 = \left(\frac{9}{4} + \frac{9}{8}\right)aT^2$
$= \frac{27}{8}aT^2$
$\left(< \frac{9aT^2}{2} = |PQ|\right)$

$\therefore$ Can reach $3aT$.

Motion is:



t_1
$v = u + at_1$
$3aT = 0 + 2at_1$
$\frac{3T}{2} = t_1$

t_3
$v = u + at$
$0 = 3aT - 4at_3$
$\frac{3T}{4} = t_3$

Find t_2:

Dist at constant speed:

$\frac{9aT^2}{8} - \frac{27}{8}aT^2 = \frac{9aT^2}{8}$

$\therefore s = ut$

$\Rightarrow \frac{9aT^2}{8} = 3aTt_2 \Rightarrow t_2 = \frac{3T}{8}$

Total time $= \frac{3}{2}T + \frac{3}{8}T + \frac{3}{4}T$.
$= \left(\frac{12+3+6}{8}\right)T$
$= \frac{21}{8}T$ secs.

(ii) Top speed $5aT$.

? Can he reach $5aT$.

Accel

$v = 5aT$
$u = 0$
$a = 2a$
$v^2 = u^2 + 2as_1$
$\Rightarrow \frac{25aT^2}{4} = s_1$

Decel

$u = 5aT$
$v = 0$
$a = -4a$
$v^2 = u^2 + 2as_3$
$\Rightarrow 0^2 = (5aT)^2 - 8as_3$
$\frac{25aT^2}{8} = s_3$

$s_1 + s_3 = \frac{25aT^2}{4} + \frac{25aT^2}{8} = \frac{75aT^2}{8}$.
$> \frac{9aT^2}{2}$

So can't reach $5aT$.

Motion is:

max speed V

T = total time

Total dist $= \frac{1}{2}VT \Rightarrow \boxed{\frac{9aT^2}{2} = \frac{1}{2}VT}$

But $2a : 4a = t_2 : t_1$
$\Rightarrow 1 : 2 = t_2 : t_1 \Rightarrow \boxed{t_1 = \frac{2}{3}T}$

$\therefore v = u + at \Rightarrow V = 0 + 2a\left(\frac{2}{3}T\right)$

$\therefore \frac{9aT^2}{2} = \frac{1}{2}\left(2a\left(\frac{2}{3}T\right)\right) \cdot T$

$\frac{27T^2}{4} = T^2$

$\boxed{\sqrt{\frac{27}{4}}T = T}$